Fermion Scattering off Electroweak Bubble Wall and Baryogenesis

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ABSTRACT

By treating CP-violating interaction as a perturbative term, we solve the Dirac equation in the background of electroweak bubble wall (the distorted wave Born approximation). We obtain the transmission and reflection coefficients for a chiral fermion incident from the symmetric-phase region and for the one from the broken-phase region respectively. There hold the respective sets of unitarity relations and also reciprocity relations among them. These relations enable us to rigorously derive quantum-number flux through the bubble wall, which is the first order quantity of the CP violation. The flux is found to be negligible for a thick wall such that $m_0/a \gtrsim 2$, where 1/a is the wall thickness and m_0 is the fermion mass.

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1. Introduction

It is well known that electroweak theory satisfies the three necessary conditions by Sakharov¹⁾ to generate the baryon asymmetry of the universe, provided that the phase transition is first order²⁾. During the phase-transition process of first order, we assign a complex mass to a fermion as a function of z, $m(z) = m_R(z) + im_I(z)$, where z is the coordinate perpendicular to the wall. The real part $m_R(z)$ asymptotically behaves such that $m_R(z) \to 0$ as $z \to -\infty$ (symmetric phase) and $m_R(z) \to m_0$ as $z \to +\infty$ (broken phase), where m_0 is the fermion mass. The imaginary part $m_I(z)$ produces quantum-number flow through the bubble wall³⁾, if $m_I(z)/m_R(z)$ is not a constant as in the case that CP is violated in the Higgs sector.

We give a general prescription to treat fermion propagation in CP-violating bubble-wall background, by regarding the CP-violating term as a small perturbation (DWBA—the distorted wave Born approximation)⁴⁾. We obtain the transmission and reflection coefficients for chiral fermions incident from the symmetric-phase or from the broken-phase region. There hold the respective sets of unitarity relations and also reciprocity relations among the coefficients. These relations enable us to rigorously derive quantum-number flux through the bubble wall, which is the first order quantity of the CP violation. Here the dynamical quantity, $\Delta R \equiv R_{R\to L}^s - \bar{R}_{R\to L}^s$, where $R_{R\to L}^s$ ($\bar{R}_{R\to L}^s$) is the reflection coefficient for right-handed chiral fermion (anti-fermion) incident from the symmetric-phase region, is of primary importance. If $|\Delta R|$ is small, no sufficient amount of baryon-number asymmetry of the universe is generated^{4,5)}.

Taking $m_R(z)$ of the kink type⁶⁾, we evaluate ΔR for several forms of $m_I(z)$. $|\Delta R|$ is extremely small for a thick wall such that the thickness 1/a is larger than $2/m_0$. For one-Higgs-doublet models with $1/a \sim (20-40)/T^{-7}$, this excludes the top quark from the baryogenesis game.

2. DWBA to CP-Violating Dirac Equation

2.1. Dirac equation and Ansatz

We consider one-flavor model described by the lagrangian,

$$\mathcal{L} = \bar{\psi}_L i \partial \psi_L + \bar{\psi}_R i \partial \psi_R + (f \bar{\psi}_L \psi_R \phi + \text{h.c.}). \tag{2.1}$$

In the vacuum, near the first-order phase transition point, $\langle \phi \rangle$ may be x-dependent field, so that we put

$$m(\mathbf{x}) = -f\langle \phi \rangle(\mathbf{x}),\tag{2.2}$$

where $m(\mathbf{x})$ is complex-valued and we neglect the time dependence. If the phase of $m(\mathbf{x})$ has no spatial dependence, it is removed by a constant bi-unitary transformation, which is outside of our interest. The Dirac equation describing fermion propagation in the bubble-wall background with CP violation is³⁾

$$i\partial \psi(t, \boldsymbol{x}) - m(\boldsymbol{x})P_R\psi(t, \boldsymbol{x}) - m^*(\boldsymbol{x})P_L\psi(t, \boldsymbol{x}) = 0.$$
(2.3)

For the bubble wall with large enough radius, $m(\mathbf{x})$ could be regarded as a function of only one spatial coordinate, so that we put $m(\mathbf{x}) = m(z)$.

To solve (2.3), we take the following Ansatz⁶:

$$\psi(t, \boldsymbol{x}) = (i\boldsymbol{\partial} + m^*(z)P_R + m(z)P_L)e^{i\sigma(-Et+\boldsymbol{p}_T\boldsymbol{x}_T)}\psi_E(\boldsymbol{p}_T, z)$$

$$= e^{i\sigma(-Et+\boldsymbol{p}_T\boldsymbol{x}_T)}[\sigma(\gamma^0 E - \gamma_T p_T) + i\gamma^3 \partial_z + m^*(z)P_R + m(z)P_L]\psi_E(\boldsymbol{p}_T, z),$$
(2.4)

where $\sigma = +(-)$ for positive (negative)-energy solution, $\mathbf{p}_T = (p^1, p^2)$, $\mathbf{x}_T = (x^1, x^2)$, $p_T = |\mathbf{p}_T|$ and $\gamma_T p_T = \gamma^1 p^1 + \gamma^2 p^2$. By putting $E = E^* \cosh \eta$ and $p_T = E^* \sinh \eta$ with $E^* = \sqrt{E^2 - p_T^2}$, the Lorentz transformation eliminates \mathbf{p}_T . After this Lorentz rotation for a fixed \mathbf{p}_T , the Dirac equation is rewritten as

$$\left[E^{*2} + \partial_z^2 - |m(z)|^2 + im_R'(z)\gamma^3 - m_I'(z)\gamma_5\gamma^3\right]\psi_E(z) = 0, \tag{2.5}$$

where $m(z) = m_R(z) + im_I(z)$. Now let us introduce a set of dimensionless variables using a parameter a, whose inverse characterizes the thickness of the wall: $m_R(z) =$

 $m_0 f(az) = m_0 f(x), m_I(z) = m_0 g(az) = m_0 g(x), x \equiv az, \tau \equiv at, \epsilon \equiv E^*/a, \xi \equiv m_0/a,$ where m_0 is the fermion mass in the broken phase. Eq.(2.5) is expressed as

$$\left[\epsilon^2 + \frac{d^2}{dx^2} - \xi^2 (f(x)^2 + g(x)^2) + i\xi f'(x)\gamma^3 - \xi g'(x)\gamma_5\gamma^3\right]\psi_{\epsilon}(x) = 0.$$
 (2.6)

As for f(x) and g(x), we do not specify their functional forms but only assume that

$$f(x) \to \begin{cases} 1, & \text{as } x \to +\infty, \\ 0, & \text{as } x \to -\infty, \end{cases}$$
 (2.7)

and that |g(x)| << 1, *i.e.*, small CP violation. Eq.(2.7) means that the system is in the broken (symmetric) phase at $x \sim +\infty$ ($x \sim -\infty$), the wall height being m_0 .

2.2. DWBA TO THE DIRAC EQUATION $^{4)}$

We regard the small |g(x)| as a perturbation, and keep quantities up to $O(g^1)$. Put

$$\psi_{\epsilon}(x) = \psi^{(0)}(x) + \psi^{(1)}(x), \tag{2.8}$$

where $\psi^{(0)}(x)$ is a solution to the unperturbed equation

$$\left[\epsilon^2 + \frac{d^2}{dx^2} - \xi^2 f(x)^2 + i\xi f'(x)\gamma^3\right]\psi^{(0)}(x) = 0$$
(2.9)

with an appropriate boundary condition. Then $\psi^{(1)}(x)$ of $O(g^1)$ is solved as

$$\psi^{(1)}(x) = \int dx' G(x, x') V(x') \psi^{(0)}(x') \quad \text{with} \quad V(x) = -\xi g'(x) \gamma_5 \gamma^3. \quad (2.10)$$

G(x, x') is the Green's function for the operator in (2.9) satisfying the same boundary condition as $\psi^{(0)}(x)$. To this order, the solution to the Dirac equation is given by

$$\psi(x) \simeq e^{-i\sigma\epsilon\tau} \left\{ [\sigma\epsilon\gamma^0 + i\gamma^3 \frac{d}{dx} + \xi f(x)] [\psi^{(0)}(x) + \psi^{(1)}(x)] - i\xi g(x)\gamma_5\psi^{(0)}(x) \right\}.$$
 (2.11)

If we expand $\psi^{(0)}(x)$ in terms of the eigenspinors of γ^3 as $\psi^{(0)}(x) \sim \phi_{\pm}(x)u_{\pm}^s$ with

 $\gamma^3 u_{\pm}^s = \pm i u_{\pm}^s (s=1,2), \ \phi_{\pm}(x)$ satisfies

$$\left[\epsilon^2 + \frac{d^2}{dx^2} - \xi^2 f(x)^2 \mp \xi f'(x)\right] \phi_{\pm}(x) = 0. \tag{2.12}$$

Because of (2.7), the asymptotic forms of $\phi_{\pm}(x)$ should be $\phi_{\pm}(x) \to e^{\alpha x}$, $e^{-\alpha x}(x \to +\infty)$ and $e^{\beta x}$, $e^{-\beta x}(x \to -\infty)$, where $\alpha = i\sqrt{\epsilon^2 - \xi^2}$ and $\beta = i\epsilon$. Putting all these together, we obtain the asymptotic forms of the wave function (2.11) at $x \to \pm \infty$.

2.3. Fermion incident from symmetric-phase $\operatorname{region}^{4)}$

We consider a state in which the incident wave coming from $x=-\infty$ is reflected in part at the bubble wall, while at $x=+\infty$ only the transmitted wave exists. We denote two independent solutions to (2.12) as $\phi_{\pm}^{(+\alpha)}(x)$ and $\phi_{\pm}^{(-\alpha)}(x) = (\phi_{\pm}^{(+\alpha)}(x))^*$ which behave as

$$\phi_{\pm}^{(+\alpha)}(x) \to e^{\alpha x}, \qquad \phi_{\pm}^{(-\alpha)}(x) \to e^{-\alpha x}$$
 (2.13)

at $x \to +\infty$. Their asymptotic forms at $x \to -\infty$ are

$$\phi_{\pm}^{(+\alpha)}(x) \sim \gamma_{\pm}(\alpha, \beta) e^{\beta x} + \gamma_{\pm}(\alpha, -\beta) e^{-\beta x},$$

$$\phi_{+}^{(-\alpha)}(x) \sim \gamma_{+}(-\alpha, \beta) e^{\beta x} + \gamma_{+}(-\alpha, -\beta) e^{-\beta x}.$$
(2.14)

From these, the general solution to (2.9) is eventually given as

$$\psi^{(0)}(x) = \sum_{s} [A_s^{(-)} \phi_+^{(-\alpha)}(x) + A_s^{(+)} \phi_+^{(+\alpha)}(x)] u_+^s.$$
 (2.15)

The required boundary condition is achieved by setting $A_s^{(-)}=0$ for $\sigma=+$ and $A_s^{(+)}=0$ for $\sigma=-$ respectively. The Green's function which matches this boundary condition can be constructed from $\phi_{\pm}^{(\pm\alpha)}(x)$.

From the asymptotic forms of $(\psi_{\sigma}(x))^{inc}$, $(\psi_{\sigma}(x))^{trans}$ and $(\psi_{\sigma}(x))^{refl}$ in (2.11), we obtain those of the vector and axial-vector currents, $j_V^{\mu} = \bar{\psi}\gamma^{\mu}\psi$ and $j_A^{\mu} = \bar{\psi}\gamma^{\mu}\gamma_5\psi$.

In terms of the chiral currents, $j_L^{\mu}=(1/2)(j_V^{\mu}-j_A^{\mu})$ and $j_R^{\mu}=(1/2)(j_V^{\mu}+j_A^{\mu})$, the transmission and reflection coefficients for the chiral fermion are defined as

$$T_{L\to L(R)}^{(\sigma)} = \left(j_{L(R),\sigma}^{3}\right)^{trans} \Big|_{A_{1}^{\sigma}=0} / \left(j_{L,\sigma}^{3}\right)^{inc},$$

$$T_{R\to L(R)}^{(\sigma)} = \left(j_{L(R),\sigma}^{3}\right)^{trans} \Big|_{A_{2}^{\sigma}=0} / \left(j_{R,\sigma}^{3}\right)^{inc},$$

$$R_{L(R)\to R(L)}^{(\sigma)} = -\left(j_{R(L),\sigma}^{3}\right)^{refl} / \left(j_{L(R),\sigma}^{3}\right)^{inc}.$$
(2.16)

If we denote $R^s(T^s) = R^{(+)}(T^{(+)})$ and $\bar{R}^s(\bar{T}^s) = R^{(-)}(T^{(-)})$, where the superscript s denotes the fermion incident from the symmetric-phase region, we have

$$T_{L\to L}^{s} = \bar{T}_{R\to R}^{s} = \frac{\sqrt{\epsilon^{2} - \xi^{2}} + \epsilon}{2\epsilon |\gamma_{+}(\alpha, \beta)|^{2}} (1 - \delta^{inc}),$$

$$T_{L\to R}^{s} = \bar{T}_{R\to L}^{s} = \frac{\sqrt{\epsilon^{2} - \xi^{2}} - \epsilon}{2\epsilon |\gamma_{+}(\alpha, \beta)|^{2}} (1 - \delta^{inc}),$$

$$T_{R\to L}^{s} = \bar{T}_{L\to R}^{s} = \frac{\sqrt{\epsilon^{2} - \xi^{2}} - \epsilon}{2\epsilon |\gamma_{+}(\alpha, \beta)|^{2}} (1 + \delta^{inc}),$$

$$T_{R\to R}^{s} = \bar{T}_{L\to L}^{s} = \frac{\sqrt{\epsilon^{2} - \xi^{2}} + \epsilon}{2\epsilon |\gamma_{+}(\alpha, \beta)|^{2}} (1 + \delta^{inc}),$$

$$R_{L\to R}^{s} = \bar{R}_{R\to L}^{s} = \left| \frac{\gamma_{+}(\alpha, -\beta)}{\gamma_{+}(\alpha, \beta)} \right|^{2} (1 - \delta^{inc} - \delta^{refl}),$$

$$R_{R\to L}^{s} = \bar{R}_{L\to R}^{s} = \left| \frac{\gamma_{+}(\alpha, -\beta)}{\gamma_{+}(\alpha, \beta)} \right|^{2} (1 + \delta^{inc} + \delta^{refl}).$$

Here the corrections by the CP violation are

$$\delta^{inc} = \frac{\xi}{2\sqrt{\epsilon^2 - \xi^2}} \left(\frac{\gamma_-(-\alpha, \beta)}{\gamma_+(\alpha, \beta)} I + c.c. \right),$$

$$\delta^{refl} = \frac{\xi}{2\sqrt{\epsilon^2 - \xi^2}} \left(\frac{\gamma_-(-\alpha, -\beta)}{\gamma_+(\alpha, -\beta)} I + c.c. \right),$$
(2.18)

where $I = \int_{-\infty}^{\infty} dx \, g'(x) \phi_{-}^{(+\alpha)}(x) \phi_{+}^{(+\alpha)}(x)$. Among these, the following unitarity relations hold:

$$T_{L\to L}^s + T_{L\to R}^s + R_{L\to R}^s = 1, \qquad T_{R\to L}^s + T_{R\to R}^s + R_{R\to L}^s = 1.$$
 (2.19)

2.4. Fermion incident from Broken-Phase region and reciprocity $^{5)}$

In place of $\phi_{+}^{(+\alpha)}(x)$, we start with $\phi_{+}^{(-\beta)}(x)$:

$$\phi_{\pm}^{(-\beta)}(x) \to e^{-\beta x}, \qquad \phi_{\pm}^{(+\beta)}(x) = (\phi_{\pm}^{(-\beta)}(x))^* \to e^{+\beta x}$$
 (2.20)

at $x \to -\infty$, while at $x \to +\infty$

$$\phi_{\pm}^{(-\beta)}(x) \sim \widetilde{\gamma}_{\pm}(-\beta, \alpha) e^{\alpha x} + \widetilde{\gamma}_{\pm}(-\beta, -\alpha) e^{-\alpha x},$$

$$\phi_{\pm}^{(+\beta)}(x) \sim \widetilde{\gamma}_{\pm}(\beta, \alpha) e^{\alpha x} + \widetilde{\gamma}_{\pm}(\beta, -\alpha) e^{-\alpha x}.$$
(2.21)

In parallel to Sect.2.3, we can derive the transmission and reflection coefficients (denoted by the superscript b) such as $T^b_{L\to L} = \bar{T}^b_{R\to R}$ and $R^b_{L\to L} = \bar{R}^b_{R\to R} = -R^b_{R\to R} = -\bar{R}^b_{L\to L}$. The following unitarity relations hold as they should:

$$T_{L\to L}^b + T_{L\to R}^b + R_{L\to L}^b + R_{L\to R}^b = 1, \qquad T_{R\to L}^b + T_{R\to R}^b + R_{R\to L}^b + R_{R\to R}^b = 1.$$
 (2.22)

Since $\phi_{\pm}^{(\pm\beta)}(x)$ are linearly dependent on $\phi_{\pm}^{(\pm\alpha)}(x)$, we have $\widetilde{\gamma}_{\pm}(\beta,\alpha) = (\beta/\alpha)\gamma_{\pm}(-\alpha,-\beta)$ and so on. Then we can prove reciprocity relations for the chiral fermion scattered off the bubble wall including CP violation of $O(g^1)$:

$$T_{L\to R}^b + T_{R\to R}^b = T_{L\to L}^s + T_{L\to R}^s, \qquad T_{L\to L}^b + T_{R\to L}^b = T_{R\to L}^s + T_{R\to R}^s.$$
 (2.23)

3. Quantum-Number Flux through Bubble Wall⁵⁾

Let Q_L (Q_R) be an additive quantum number carried by the left(right)-handed fermion, which is conserved in the symmetric phase. The quantum-number flux into the symmetric-phase region just in front of the bubble wall moving with velocity u is given by

$$F_{Q} = \frac{1}{\gamma} \int_{m_{0}}^{\infty} dp_{L} \int_{0}^{\infty} \frac{dp_{T}p_{T}}{4\pi^{2}} (Q_{L} - Q_{R})$$

$$\times \left[(R_{R \to L}^{s} - \bar{R}_{R \to L}^{s}) f^{s}(p_{L}, p_{T}) - (T_{L \to R}^{b} + T_{R \to R}^{b} - T_{L \to L}^{b} - T_{R \to L}^{b}) f^{b}(-p_{L}, p_{T}) \right], \tag{3.1}$$

where $\gamma = \sqrt{1 - u^2}$ and the fermion-flux density in the symmetric (broken) phase $f^s(f^b)$ is given by

$$f^{s}(p_{L}, p_{T}) = (p_{L}/E) \left(\exp\left[\gamma(E - up_{L})/T\right] + 1\right)^{-1},$$

$$f^{b}(-p_{L}, p_{T}) = (p_{L}/E) \left(\exp\left[\gamma(E + u\sqrt{p_{L}^{2} - m_{0}^{2}})/T\right] + 1\right)^{-1},$$
(3.2)

the chemical potential being omitted for simplicity.

Thanks to the unitarity and reciprocity relations, (3.1) is reduced to a simple expression:

$$F_Q = \frac{1}{\gamma} \int_{m_0}^{\infty} dp_L \int_0^{\infty} \frac{dp_T p_T}{4\pi^2} (Q_L - Q_R) \Big[f^s(p_L, p_T) - f^b(-p_L, p_T) \Big] \Delta R.$$
 (3.3)

Here ΔR is the difference between the chiral fermion and its anti-fermion in the reflection coefficients incident from the symmetric-phase region:

$$\Delta R \equiv R_{R \to L}^s - \bar{R}_{R \to L}^s = 2R^{(0)s}(\delta^{inc} + \delta^{refl}) = -2T^{(0)s}\delta^{inc}, \tag{3.4}$$

where

$$T^{(0)s} = \frac{\alpha}{\beta} \frac{1}{|\gamma_{+}(\alpha, \beta)|^{2}}, \qquad R^{(0)s} = \left| \frac{\gamma_{+}(\alpha, -\beta)}{\gamma_{+}(\alpha, \beta)} \right|^{2}$$
(3.5)

with $T^{(0)s} + R^{(0)s} = 1$ are respectively the transmission and reflection coefficients in the absence of CP violation, and we have used $\delta^{inc} + R^{(0)}\delta^{refl} = 0$.

We now recognize that ΔR is the dynamical quantity of primary importance, since, if its absolute value be small, any quantum-number flow is almost forbidden. Note that the vector-like quantum numbers such as baryon and lepton numbers do not flow through the CP-violating bubble wall. (The next order correction would be $O(g^3)$.) Then one might have to resort to some charge transport scenario or other to explain cosmological baryogenesis^{3,8}. If one takes hypercharge flux, for example, $(Q_L - Q_R) = -1, 1, 1$ for the up-type quark, down-type quark, massive lepton respectively.

4. Quantum-Number Flux through the Kink-Type Bubble Wall

A simple profile of the bubble wall without CP violation may be of the kink type studied in Ref.6):

$$f(x) = (1 + \tanh x)/2. \tag{4.1}$$

The unperturbed solutions $\phi_{\pm}^{(\pm\alpha)}(x)$ are expressed in terms of the hypergeometric functions. The transmission and reflection coefficients without CP violation are respectively

$$T^{(0)s} = \frac{\sin(\pi\alpha)\sin(\pi\beta)}{\sin[\frac{\pi}{2}(\alpha+\beta+\xi)]\sin[\frac{\pi}{2}(\alpha+\beta-\xi)]},$$

$$R^{(0)s} = \frac{\sin[\frac{\pi}{2}(\alpha-\beta+\xi)]\sin[\frac{\pi}{2}(\alpha-\beta-\xi)]}{\sin[\frac{\pi}{2}(\alpha+\beta+\xi)]\sin[\frac{\pi}{2}(\alpha+\beta-\xi)]}.$$
(4.2)

The effects of CP violation can be evaluated, once the functional form of g(x) is given. We have executed numerical calculations for several forms of $g(x)^{5}$. We show one example in Fig.1,* i.e., $g(x) = \Delta\theta \cdot f(x)^2$, where $\Delta\theta$ characterizes the magnitude of CP violation. (Note that $g(x) = \Delta\theta \cdot f(x)$ gives rise to no quantum-number flow, as the x-independent phase is removed away.) Although the sign of $\Delta R/\Delta\theta$ varies depending on functional forms of g(x), the dependence of $\Delta R/\Delta\theta$ on the wall thickness 1/a shows a remarkable general trend. Namely, $|\Delta R/\Delta\theta|$ is very small for $a/m_0 \lesssim 0.5$, as typically illustrated in Fig.1. Then it rapidly grows as a increases but turns to decrease for $a/m_0 \gtrsim 2$.

^{*} Because of a trivial error in our program for numerical calculation, Figures in Ref.4) are incorrect.

5. Concluding Remarks

Although many complicated factors such as the wall velocity u would largely affect baryogenesis, a sufficient amount of baryon-number asymmetry could not be produced unless $|\Delta R/\Delta \theta|$ due to CP violation is large enough. We conclude with some remarks.

- (1) The general trend mentioned just before would require that, in order for an effective baryogenesis to work, the wall thickness 1/a is smaller than $2/m_0$, about twice the Compton wave length of the relevant fermion. This would imply that, for 1/a = C/T with $C \simeq (20-40)$ of one-Higgs-doublet models⁷⁾, any fermion whose mass m_0 is subject to $m_0 \gtrsim 2T/C$ could not contribute to baryogenesis. For $T \sim 200$ GeV, the top quark is excluded from the baryogenesis game, even though its interaction with the bubble wall is large. A larger Yukawa coupling does not necessarily mean a larger effect of CP violation.
- (2) It is impossible for the one-Higgs-doublet model to incorporate x-dependent phase into the bubble wall. On the other hand, there would be many varieties of two-Higgs-doublet models or SUSY extension of the standard model, among which a new source of CP violation g(x) may be incorporated. Once the wall thickness 1/a is given, it selects what species of fermions take part in baryogenesis game.
- (3) If some of the models predict a thin bubble wall, they may generally be favored to explain the cosmological baryogenesis. We expect that our DWBA prescription and the results would serve to build models to generate baryon asymmetry of the universe.

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Figure Caption

Fig.1 $\Delta R/\Delta \theta$ as a function of E^* for various a, in the case where $g(x) = \Delta \theta \cdot f(x)^2$. The numerical values of E^* and a are respectively those of E^*/m_0 and a/m_0 .

This figure "fig1-1.png" is available in "png" format from:

http://arXiv.org/ps/hep-ph/9405422v1

